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ABSTRACT

In every school there are bright, motivated students who experience success in every field of academic endeavor except mathematics. These students are defined as dyscalculics, or learning disabled in mathematics. Piaget's model for cognitive development was used in a study of students evidencing this condition. The purpose of the study was to compare cognitive development as measured by the ability to conserve length, area, and volume in two groups of seventh-grade students. The groups consisted of students of average or above average intelligence who (1) evidenced learning disability in mathematics but who performed well in reading (N=30) and (2) those who performed well in both subjects (N=30). Subjects were selected from seventh-graders in the Shelby County, Alabama, school system. A comparison of selected factors indicated different maturation levels between the dyscalculic and control groups. Results indicate that the proportion of students who could conserve volume in the dyscalculic group to be significantly less than the proportion of conservers in the control group. Further, sex differences were found among control pupils with nearly 89 percent of the males but only 50 percent of the females capable of conserving volume. Data are seen to support previous findings that conservation is essential to mathematical success. (Author/MP)

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Cognitive Development in Students Evidencing Dyscalculia

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Introduction

In every school there are bright, motivated students who experience success in every field of academic endeavor except mathematics. These students are easily recognizable -- they like everything but math. By the time they reach high school most of them have become math avoiders, a phenomenon that greatly limits their educational and career choices.

Even though lack of achievement in capable students is generally equated with lack of motivation, that educational axiom is frequently inaccurate with these students. Most are highly motivated and have a great need to achieve. Many of these intelligent, challenging, but frustrating students are learning disabled in mathematics. They are defined operationally as dyscalculics. Dyscalculia is a disorder of the ability for learning mathematics that does not affect general intelligence. This learning disability is not limited to isolated cases: Preliminary data indicate that approximately 6% of our student population is so afflicted (Kosc, 1974; Weinstein, 1978; Tishler, 1981).

However, there has been little attention given to dyscalculia. While a parallel learning disability, dyslexia, has received widespread recognition and has generated much research, dyscalculia has not. This is indeed a paradox in a technological society such as ours that relies extensively on the application of mathematics. One explanation of this paradox appears to be that low achievement in mathematics is

more socially acceptable (particularly for girls!) than low achievement in reading.

Piaget's theory of cognitive development has been extensively used in studying the mathematical ability of children. His theory posits a model of cognitive development that is based on the acquisition of sucessive levels or stages of intellectual thought. Each level moves the child closer to formal, logical thought. Because the stages are sequential, the acquisition of one stage is prerequisite to the acquisition of the next.

The length of time required to move from one stage to the next may vary with individuals, but the sequence of the stages is invariant (Piaget, 1950). These stages, broadly summarized, are as follows:

1. Sensori-motor stage (0-2 years). Motor behavior characterizes this stage. Conceptual thought is absent though cognitive development is seen.

2. Preoperational stage (2-7 years). This stage is characterized by language development and conceptual development; however, thinking is perception bound.

3. Concrete operational stage (7-11 years). The gr dual acquisition of logical thought is applied to concrete problems at this stage.

4. Formal operational stage (11 years and above). Logical thought is applied to all types of problems, both concrete and abstract (Piaget, 1950).

A characteristic that is acquired gradually during the concrete operational stage, the concept of conservation, is central to Piaget's theory of cognitive development. Conservation of quantity (number, length, mass, area, volume) is the ability to conceive the sameness or invariance of quantity despite irrelevant changes in shape or position. Piaget wrote: "Our contention is merely that conservation is a necessary condition for all rational activity" (Piaget, 1952, p. 3). This ability indicates the child's reliance on logical thought rather than visual perception. Piaget used the acquisition of conservation as an indicator of the progressive movement toward logical thought in the concrete operational stage and believed that the ability to conserve volume, the last conservation attained, is the hallmark of the final phase of this stage (Piaget, 1950).

The use of Piaget's model for cognitive development was used to investigate the cognitive development of students evidencing dyscalculia.

Statement of the Problem

The purpose of this study was to compare cognitive development as measured by the ability to conserve length, area, and volume in two groups of seventh grade students: (1) children of average or above average IQ who evidence learning disability in mathematics but who perform well in reading and (2) children of average or above av erage IQ who perform well in both mathematics and reading.

Methodology

Subjects

Subjects were selected from seventh-grade students in the Shelby County Alabama School System. Group membership was determined by students' IQ scores, by reading and mathematics scores on the California Achievement Test (CAT), and by consultation with the mathematics teachers. For the purposes of the study, students whose CAT grade equivalent mathematics scores were 6.2 or below, whose CAT reading scores were 8.0 or above, and whose IQ estimates were 95 to 125 were identified as dyscalculic. One group was comprised of students evidencing dyscalculia. The Control Group was comprised of students whose IQ and reading scores were similar to those in the Dyscalculic Group, but who were performing at least one year above grade level in mathematics. Each group contained 30 students.

Instrumentation

Piaget developed several tests that enable a child to demonstrate conservation reasoning. Numerous validation studies conducted with these tests have supported Piaget's original conclusions (e.g., Elkind, 1961a, 1961b; Modgil, 1974; Steffe, 1968). Replication of these Piagetian tests were administered individually to each student in this study to measure conservation of length, area, and volume. Ability to conserve number was not assessed since no significant difference was found between younger (fifth grade) dyscalculics

and their controls in a prior study due to a "a pervasive ceiling effect" (Weinstein, 1978, p. 195). The following are brief descriptions of these tests.

Conservation of length test. The materials for this test consisted of two strings of beads of identical length, a plastic horse, and two blocks of wood. The examiner placed the strings of beads side by side so that the ends matched. The horse was placed at one end of the beads and a block of wood was placed at the other end of each string of beads. The student was told that the length of each string of beads was the same and was asked if the horse would have to walk the same distance down either of the roads to reach the hay. This question has been designated the "identity question" (Elking, 1961a). After agreement that each distance was the same, the student was asked if each road would remain the same distance if the examiner bent one string of beads. This is the "prediction question" (Elkind, 1961a). The examiner then bent one string of beads and asked whether the horse would have to walk as far to the hay on one road as on the other. This is a "judgment question" (Elkind, 1961a). Finally, the student was asked to explain his/her answer. According to Elking (1961a), the four types of questions asked, identity, prediction, judgment, and explanation, will provide a measure of realiability to the responses.

Conservation of area test. The materials for this test were two sheets on one-foot square green poster board, two plastic horse, and six red cubes of identical size.

The two sheets of poster board were stacked on each other to show that they were equal in size. The horses were placed on the sheets of poster board, and the student was told to imagine that each sheet was a pasture of green grass that the horse may eat. One red cube was placed in the same position on each green sheet representing a barn. This barn covered up the same amount of grass in each pasture, and the student was asked to affirm that each horse still had the same amount of grass to eat (identity question). The student was then asked whether placing two more barns on each pasture, on one sheet placing them next to the first barn and on the other sheet spreading them around, would make the amount of grass left for each horse the same or different (predition question). After the examiner completed this placement, the student was asked this question again (judgment question). Explanations were then requested.

Conservation of volume test. Materials for this test were two identical clay balls (1 1/2 inches in diameter). The student verified that the balls were the same by examination. The following questions were asked: (1) Do both balls occupy the same volume or same space? (identity question); (2) if one ball were rolled into a sausage shape, would both balls occupy the same volume or same space? (prediction question); (3) now that one ball has been rolled into a sausage shape, do both balls occupy the same volume or space? (judgment question). Students were then asked to explain their answer.

For each of the conservation tests administered, a student was scored as a conserver of that quantity only if responses to the identity, prediction, judgment, and explanation

questions were correct. A response to the explanation question was considered correct when the concept was expressed that the initial equivalence of the two quantities had been maintained despite perceptual change.

Data Collection

The Piagetian tests for conservation of length, area, and volume were individually administered to the selected subjects by the investigator during the spring of 1981. The order of these tests were: (1) conservation of length, (2) conservation of area, and (3) conservation of volume. All tests were scored by the investigator.

Data Analysis

Group comparisons of the proportion of students who were conservers of length, who were conservers of area, and who were conservers of volume were made using the chi square statistic. Sex comparisons of the proportion of students who conserve volume were made with chi square also. Because all frequencies were small in this study, Yates correction for continuity was applied to the statistic. This was deemed advisable since the theoretical frequency curve of chi square may not provide a good fit to the data in such cases (Ferguson, 1976, p. 201). The corrected chi square was then used to determine statistical significance.

Results

The summary statistics that were used to compare salient characteristics of the two groups indicated similarities on age, IQ, and grade equivalent reading scores. Significant disparity between groups was shown on grade equivalent mathematics scores. Table 1 shows the characteristics of the sample.

Table 1

Characteristics of the Sample

Variable	Group			
	Dyscalculic		Control	
	Mean	SD	Mean	SD
Age (in months)	156.26	5.72	154.80	4.75
IQ	107.10	8.44	111.80	6.89
Grade equivalent reading score	8.7	1.33	9.0	.90
Grade equivalent mathematics score	5.6	.54	9.4	1.05

Conservation of Length

The ability to conserve length was determined by administering the Piagetian test for conservation of length as described previously. Correct responses to a series of four questions

were necessary for a student to be classified as a conserver of length.

All 60 students tested in the study exhibited the ability to conserve length. The data indicate a ceiling effect for this test and support Piaget's (1950) findings that this conservation ability is generally acquired prior to 12 years of age.

Conservation of Area

The ability to conserve area was assessed by administering the Piagetian test for this purpose as previously described. The chi square statistic was used to determine if groups and conservation of area were independent. Yate's correction was applied to the statistic since the minimum expected cell frequency was 3.0. Even though a greater proportion of students in the Control Group (96.7%) could conserve area than in the Dyscalculic Group (83.8%), these data yielded a nonsignificant chi square. Table 2 reports these results.

Table 2

Contingency Table Showing Relationship between Groups and Conservation of Area

	Conserve	Non-conserve	Total
DYSCALCULICS	25 (83.3%)	5 (16.7%)	30 (100%)
CONTROLS	29 (96.7%)	1 (3.3%)	30 (100%)
TOTAL	54 (90.0%)	6 (10.0%)	60 (100%)

Chi square = 1.67 (nonsignificant) p = .1967 df = 1

Conservation of Volume

The ability to conserve volume was assessed by administering the Piagetian test for conservation of volume as previously explained. The chi square statistic was used to determine if group membership and conservation of volume were independent. Yates' correction was again applied, and the corrected chi square indicated that a significant discrepancy existed between groups with regard to the ability to conserve volume.

In the Dyscalculic Group only 7 students (23.8%) were classified as conservers of volume while in the Control Group the number thus classified was more than triple that figure: 22 (73.3%) of the students in the Control Group demonstrated this ability. Even though 31 of the 60 students tested had not attained this stage of cognitive development, 23 of those nonconservers were in the Dyscalculic Group.

A significant chi square was found with these data (chi square = 13.08, df = 1, p = .0003). A contingency coefficient of .447 was computed (maximum = .707) thus indicating a moderately strong association between groups and ability to conserve volume. The contingency table showing these data appears in Table 3.

Table 3

Contingency Table Showing Relationship between Groups and Conservation of Volume

	Conserve	Non-conserve	Total
Dyscalculics	7 (23.3%)	23 (76.7%)	30 (100%)
Controls	22 (73.3%)	8 (26.7%)	30 (100%)
Total	29 (48.3%)	31 (51.7%)	60 (100%)

Chi square = 13.08 (significant) $p = .0003$)

$df = 1$

Contingency coefficient = .447

Sex differences within each group were also compared on this variable. The contingency table for the Dyscalculic Group showed a nonsignificant value for chi square. However, sex differences were notable: 91.7% of the females in the Dyscalculic Group could not conserve volume as compared to 66.7% of the males. These data are found in Table 4.

Sex differences in the Control Group for conservation of volume were statistically significant (chi square = 3.75, $df = 1$, $p = .048$). Again, a higher percentage of females demonstrated inability to conserve volume than did males.

In the female group 50.0% could not conserve volume while in the male group only 11.1% lacked this ability. Table 5 reports these results.

Table 4

Contingency Table for Dyscalculic Group Showing Relationship between Sex and Conservation of Volume

	Conserve	Non-conserve	Total
Males	6 (33.3%)	12 (66.7%)	18 (100%)
Females	1 (8.3%)	11 (91.7%)	12 (100%)
Total	7 (23.3%)	23 (76.7%)	30 (100%)

Chi square = 1.31 (nonsignificant) $p = .25$

$df = 1$

Table 5

Contingency Table for Control Group Showing Relationship between Sex and Conservation of Volume

	Conserve	Non-conserve	Total
Males	16 (88.9%)	2 (11.1%)	18 (100%)
Females	6 (50.0%)	6 (50.0%)	12 (100%)
Total	22 (73.3%)	8 (26.7%)	30 (100%)

Chi square = 3.75 (significant) $p = .048$

$df = 1$

Contingency coefficient = .396

Conclusions and Discussions

A comparison of selected factors of cognitive development indicated different maturation levels between the dyscalculic and the control groups. The ability to conserve the quantities of length, area, and volume were employed as indices of maturation to sucessively higher levels of cognitive development.

Prior research has shown consistently that a child acquires conservation of length before conservation of area, while conservation of volume is the last conservation ability to be attained (Elkind, 1961a). The results of the study supported Piaget's hierarchial model. All students demonstrated the ability to conserve length, the lowest-level ability tested. However, only 90% of the students showed conservation ability for area and less than half (48.3%) of the students had attained the ability to conserve volume. Since these students were tested at approximately 13 years of age, the results concur with Elkind's work (1961a, 1961b) that showed the acquisition of higher levels of cognitive development to occur later than Piaget had predicted, although the order of acquisition is the same.

This study found the the proportion of students who could conserve volume in the dyscalculic group was significantly ($p = .0003$) less than the proportion of conservers in the Control Group. Twenty-three of the 31 students identified as nonconservers of volume were in the dyscalculic group.

It is interesting to note that significant sex differences within the Control Group were also found for conservation of volume. While 88.9% of the males in that group could conserve volume, only 50.0% of the females demonstrated the same ability. This male advantage in cognitive development may help explain the male superiority in mathematical aptitude and ability found among mathematically gifted students of this age in the work of Benbow and Stanley (1980). Within the dyscalculic group, males were also found to have a higher proportion of conservers than females, but these results were not statistically significant.

There were group differences favoring the Control Group on the ability to conserve area also. Five of the six students who could not conserve are were in the dyscalculic group. However, this difference was statistically nonsignificant.

Piaget (1962) felt that conservation is a necessary condition for all rational activity. (p. 3). A substantial body of research has shown that the ability to conserve is essential for success in mathematics. The results of this study provide additional evidence for that conclusion. The dyscalculic student apparently attains these conservation abilities later than other students of similar age and IQ and suffers the consequences in lower mathematics achievement.

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